

Empirical State Determination of Entangled Two-Level Systems and its Relation to Information Theory

Y. Ben-Aryeh and A. Mann

Department of Physics, Technion–Israel Institute of Technology,
Haifa 32000, Israel

B. C. Sanders

Department of Physics, Macquarie University, Sydney, New South Wales 2109,
Australia

Abstract

Theoretical methods for empirical state determination of entangled two-level systems are analyzed in relation to information theory. We show that hidden variable theories would lead to a Shannon index of correlation between the entangled subsystems which is larger than that predicted by quantum mechanics. Canonical representations which have maximal correlations are treated by the use of Schmidt decomposition of the entangled states, including especially the Bohm singlet state and the GHZ entangled states. We show that quantum mechanics does not violate locality, but does violate realism.

1. Introduction

In the present paper we would like to treat the problem of empirical determination of the quantum state of entangled two-level systems. We limit here the discussion to pure quantum states and use the criterion $\rho^2 = \rho$ for pure states, where ρ is the density matrix of the system. The problem of empirical determination of quantum states was treated by Band and Park [1,2]. They treated various systems and various kinds of measurements. Since their formalism is quite general it turns out to be quite complicated. Recently there have been important developments in relation to measurements of the quantum state of light [3]. As we treat in the present article the special case of entangled two-level systems and the special case of measurements related to Pauli spin operators, we can adopt here a different, simpler formalism. Practical measurements of two-level systems depend on the specific system and can be made for example by Stern-Gerlach devices for spin- $\frac{1}{2}$ systems [4], dipole moments and inversion of population measurements for two-level atoms [5], polarization states for entangled photons [6], etc. Our general approach to measurement of entangled two-level systems is similar to others [1-3] which have treated different systems:

“In our discussion, the term quantum state refers to the ensemble not to any individual system (An ensemble may be generated, however, by taking a single system alternately prepared, measured, identically prepared, etc.); the state determination is synonymous with the determination of the density ρ ” [2].

We follow here the approach that the state function (or more generally the density matrix) represents our state of knowledge of the system [4]: “Once it is accepted

it is not surprising that new information can change the state of our knowledge. It is also clearly no difficulty that a measurement performed in one region of space can give us information about an object which is far away without this implying the transmission of influence instantaneously [4]". We assume the "epistemological" character of the wavefunction in the sense that it is a device for making statistical predictions for future experiments on the basis of our present knowledge of the system. (Interesting studies which give "ontological" meaning to the wavefunction, i.e., it exists as a real physical wave independent of our knowledge of it, were also developed [7-10]).

Recently, with new developments of experimental methods, a number of possible practical applications of quantum entangled states have been proposed, including quantum computation [11,12] and quantum teleportation [13]. Entangled states with two particles have been employed to test Bell's inequality and to rule out local-realistic descriptions of nature [14]. Entangled states with three particles, the so-called Greenberger-Horne-Zeilinger (GHZ) states [15], and with more particles [16,17] have been proposed for studying the role of quantum correlations, and there is a large current interest in carrying out experiments on such states. It is relatively easy to produce entangled states for photons, e.g., by parametric amplifiers [6], but entanglement of two atoms (massive particles) has been produced for the first time only in 1997 [18]. Various processes for the production of entangled states have been studied including, among others, the production of entangled coherent states [19]. The phenomenon of entanglement is essentially related to the problem of empirical determination of the entangled quantum states which is the main topic of the present article.

The paper is arranged as follows: In Sec. 2 we show that the assumption of hidden variables for the correlation of entangled states gives an amount of information which is incompatible with certain results obtained by QM [20]. We use here the concept of relative state introduced by Everett [21,22], and follow some of his mathematical derivations, but do not support his physical conclusions. Our main approach to empirical determination of entangled two-level states is analyzed in Sec. 3. We show in this section that there is no “quantum nonlocality” problem, at least not for the entangled two-level systems. It was already pointed out [23] “that the idea of nonlocal influencing of one particle on another when they are in space-like separated regions has neither empirical nor theoretical support”. We support this viewpoint from another perspective, and show by our analysis that QM does not violate locality but violates realism even for single particles.

2. On the Relation between the Shannon Index of Correlation and Hidden Variable Theories for Two-Component Systems

In the articles of Everett on “The Many-Worlds interpretation of quantum mechanics” [21,22], the EPR problem has been related to the concept of “relative state function”. In this description one considers a composite system $S = S_1 + S_2$, in the state ψ^S . To every state η of S_2 a state ψ_{rel}^η is associated which is called the relative state in S_1 for η in S_2 , through

$$\psi_{\text{rel}}^\eta = N \sum_i \langle \phi_i | \eta | \psi^S \rangle | \phi_i \rangle , \quad (1)$$

where $\{\phi_i\}$ is any complete orthonormal set in S_1 and N is a normalization constant. An important property of ψ_{rel}^η is its uniqueness, i.e., it is independent of the choice of the basis $\{\phi_i\}$ [21,22]. Another important property of the relative state is that ψ_{rel}^η gives the conditional expectations of all operators, conditioned by the state η in S_2 [21,22]. Everett concludes by following his analysis: “It is meaningless to ask the absolute state of a subsystem – one can only ask the state relative to a given state of the remainder system”. By following such analysis one enters into the problem of quantum correlations between two separated subsystems.

The canonical correlation, which describes the fundamental correlation between two separated subsystems S_1 and S_2 , is obtained by choosing a representation in which both reduced density matrices ρ^{S_1} and ρ^{S_2} of the subsystems are diagonal. In this representation the state ψ^S is described by the Schmidt decomposition [21,22,24]:

$$\psi^S = \sum_i a_i \zeta_i \eta_i , \quad (2)$$

where the $\{\zeta_i\}$ and $\{\eta_i\}$ constitute orthonormal sets of states for S_1 and S_2 , respectively. Any pair of operators \hat{A} in S_1 and \hat{B} in S_2 , which have as non-degenerate eigenfunctions the set $\{\zeta_i\}$ and $\{\eta_j\}$ (i.e., operators which define the canonical representation [21,22]) are “perfectly” correlated in the sense that there is a one-one correspondence between their eigenvalues. The probability for eigenvalues λ_i of \hat{A} and μ_j of \hat{B} is given by

$$P(\lambda_i \text{ and } \mu_j) = P_{ij} \quad (3)$$

The Shannon index of correlation in this representation is given by [20]:

$$I_{\text{Shann}} = \sum_{i,j} P_{ij} \log \left(\frac{P_{ij}}{P_i P_j} \right) \quad (4)$$

Classically, one can consider two random variables X and Y with a joint probability $p(x, y)$ and marginal probabilities $p(x)$ and $p(y)$. Then the mutual information $I(X, Y)$ is given by [25,26]:

$$I(X, Y) = \sum_{x,y} p(x, y) \log \left[\frac{p(x, y)}{p(x)p(y)} \right] \quad (5)$$

Quantum mechanically one considers the Shannon index of correlation [20] which is the analog of Eq. (5) but which depends on the representation of the quantum state. The canonical representation (Eq. (2)) gives the maximal Shannon index of correlation (Eq. (4)) [20-22].

One should distinguish between the Shannon index of correlation, and the quantum index of correlation – which is related to the quantum entropy. The overall state of the two-component system is described by a density operator ρ and the

states of the component systems are described by the reduced density operators ρ_a and ρ_b . Using the definition of entropy

$$S = -\text{Tr} \rho \ln \rho , \quad (6)$$

the quantum index of correlation for a two-component system is [20]:

$$I_c = S_a + S_b - S \quad (7)$$

where

$$S_a = -\text{Tr} \rho_a \ln \rho_a : \quad S_b = -\text{Tr} \rho_b \ln \rho_b \quad (8)$$

As we restrict the discussion to pure states, the entropy S is precisely zero. Barnett and Phoenix [20] derived the important result that, for a pure two-component system the observation of the Shannon index of correlation, for a certain representation, cannot provide more information than half the information contained in the quantum index of correlation. As one can easily verify, the canonical representation given by Eq. (2) gives the maximal Shannon index of correlation $I_{\text{Shann}} = \frac{1}{2}I_c = \frac{1}{2}(S_a + S_b)$, where for pure states $S = 0$, $S_a = S_b$.

We would like to show here an interesting relation between information theory and hidden variables assumption. According to hidden variables theory the correlation between the two separated systems was produced during the interaction time in the past by common hidden variables. This idea can be represented in the case of two-component system as [27]:

$$P_{ij} = \sum_{\lambda} P_{i\lambda} P_{j\lambda} ; \quad P_i = \sum_{\lambda} P_{i\lambda} ; \quad P_j = \sum_{\lambda} P_{j\lambda} \quad (9)$$

where we have assumed that the correlation P_{ij} is produced by common hidden variables λ for the two separated systems. For simplicity we assume here a summation over hidden variables λ , but the present arguments can be easily generalized to integration over any number of continuous variables λ .

Using information theory, we find that hidden variables theories lead to a refined distribution [21,22] where the original values of P_i and P_j have been resolved into a number of values $P_{i\lambda}$ and $P_{j\lambda}$. The resolution of P_{ij} follows, however, the constraint that the two separated systems have the common parameters λ .

The refinement of Eq. (4) can be described as:

$$I_{\text{Shann}}^{HV} = \sum_{ij} \sum_{\lambda} P_{ij\lambda} \ln \left(\frac{P_{ij\lambda}}{P_i P_j P_{\lambda}} \right) \quad (10)$$

where

$$\sum_{\lambda} P_{\lambda} = 1 .$$

We can now use the log sum inequality [21,25]

$$\sum_{\lambda} X_{\lambda} \ln \left(\frac{X_{\lambda}}{a_{\lambda}} \right) \geq \sum_{\lambda} X_{\lambda} \ln \left(\frac{\sum_{\lambda} X_{\lambda}}{\sum_{\lambda} a_{\lambda}} \right) \quad (11)$$

($X_{\lambda} \geq 0$, $a_{\lambda} \geq 0$ for all λ). Using Eq. (11) in Eq. (10) we get:

$$I_{\text{Shann}}^{HV} \geq \sum_{ij} \left(\sum_{\lambda} P_{ij\lambda} \right) \ln \left(\frac{\sum_{\lambda} P_{ij\lambda}}{\sum_{\lambda} P_i P_j P_{\lambda}} \right) = \sum_{ij} P_{ij} \ln \left(\frac{P_{ij}}{P_i P_j} \right) \equiv I_{\text{Shann}} \quad (12)$$

We have equality in Eq. (12) if and only if [25]:

$$P_{ij\lambda} = P_{\lambda} C_{ij} \quad (13)$$

where C_{ij} is independent of λ . This singular case in which λ does not correlate the i and j subsystems makes redundant the hidden variables assumption [Eq. (9)] and therefore may be discarded. Disregarding this singular case we get the result:

$$I_{\text{Shann}}^{\text{HV}} > I_{\text{Shann}} , \quad (14)$$

which means that the refinement by hidden variables should increase the amount of information included in the Shannon index of correlation. In particular, if the hidden variables were compatible with other observables then, for a two-component system in a pure state, measurement of the index of correlation for the Schmidt representation would provide more information than half the information contained in the quantum index of correlation. We find that hidden variable theories lead to mutual information between subsystems which is larger than that obtained by QM. A by-product of refuting the hidden variables theories is reestablishing the quantum limit of mutual information.

3. Empirical State Determination of Entangled Two-Level Systems

An observable A is represented by an Hermitian operator A on Hilbert space H . The mean value of A , obtained from an ensemble of systems all prepared identically (described by the density matrix ρ), is given by

$$\langle A \rangle = \text{Tr}(\rho A) \quad (15)$$

The problem is to find a set of operators A so that equations (15) can be solved uniquely for ρ [1,2]. In order to determine the density operator of N two-level systems, $2^{2N} - 1$ real numbers are required (because ρ is hermitian and satisfies

$\text{Tr}\rho = 1$). We need therefore $2^{2N} - 1$ independent observables by which we can determine ρ (and for pure states we have the additional constraint $\rho^2 = \rho$).

As is well known [5], the density operator for a two-level system ($N = 1$), corresponding to a wavefunction $C_1|1\rangle + C_2|2\rangle$, can be determined by three measurements: 1) Real part of the “complex dipole” $D_1 = C^*C_2 + C_2^*C_1$. 2) Imaginary part of the complex dipole $D_2 = -i(C_1^*C_2 - C_2^*C_1)$. 3) Inversion of population $D_3 = C_2^*C_2 - C_1^*C_1$. The condition $\rho^2 = \rho$ for pure states then gives the well known relation $D_1^2 + D_2^2 + D_3^2 = 1$.

For two-spin- $\frac{1}{2}$ systems denoted by a and b we can use the Hilbert-Schmidt (HS) representation of the density operator [28]:

$$\rho = \frac{1}{4} \left\{ (I)_a \otimes (I)_b + (\vec{r} \cdot \vec{\sigma})_a \otimes (I)_b + (I)_a \otimes (\vec{s} \cdot \vec{\sigma})_b + \sum_{m,n=1}^3 t_{nm} (\sigma_n)_a \otimes (\sigma_m)_b \right\} \quad (16)$$

Here I stands for the unit operator, \vec{r}, \vec{s} belong to \mathbb{R}^3 , $\{\sigma_n\}$ ($n = 1, 2, 3$) are the standard Pauli matrices. The coefficients $t_{mn} = \text{Tr}(\rho \sigma_n \otimes \sigma_m)$ form a real matrix denoted by T . In order to obtain $(r_i)_a$ or $(s_j)_b$ one needs to perform a measurement on one corresponding arm of the measuring device, while the parameters t_{mn} involve correlation measurements which are performed on the two arms. The set of 15 real numbers separates into two different classes: 6 real numbers corresponding to \vec{r} and \vec{s} describing local properties of the entangled state, and 9 real numbers corresponding to the matrix T describing the EPR correlations [28]. If we have only the information that our system is composed of two two-level subsystems, we need to measure the expectation values of the above 15 observables in order to determine the quantum state. However, if we have, for example, the previous information that our entangled

quantum state is given by

$$|\psi\rangle = C_1|1\rangle_a|2\rangle_b + C_2|2\rangle_a|1\rangle_b, \quad (17)$$

and only the complex numbers C_1 and C_2 are not known (but $|C_1|^2 + |C_2|^2 = 1$), then the number of real numbers which should be determined by the quantum measurements is reduced to 3. The assumption made in the use of Eq. (17) is equivalent to having the information $r_i (i = 1, 2, 3) = 0$, $s_j (j = 1, 2, 3) = 0$, $t_{ij} (i \neq j) = 0$, remaining with only the unknown parameters t_{11}, t_{22}, t_{33} (where for pure states $t_{11}^2 + t_{22}^2 + t_{33}^2 = 3$). The description of the Bohm singlet state and its relation to quantum measurement has played a major role in the interpretation of EPR correlations [7]. The wavefunction of this state is a special case of Eq. (17) in which $C_1 = \frac{1}{\sqrt{2}}$, $C_2 = \frac{-1}{\sqrt{2}}$. The HS representation of the density matrix of this state is given by the following sum of direct products [24]:

$$\rho = \frac{1}{4}(I)_a \otimes (I)_b - \frac{1}{4}(\sigma_1)_a \otimes (\sigma_1)_b - \frac{1}{4}(\sigma_2)_a \otimes (\sigma_2)_b - \frac{1}{4}(\sigma_3)_a \otimes (\sigma_3)_b \quad (18)$$

Here a straightforward calculation of the HS parameters gives the values $t_{11} = t_{22} = t_{33} = -1$, and all other parameters are equal to zero. One should take into account that quantum information is included also in the parameters which are equal to zero. A straightforward calculation for the density matrix of the Bohm singlet state gives

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (19)$$

Although all the quantum information of the Bohm singlet state is included in Eq. (19), a more direct relation to quantum measurements and physical insight into the EPR problem is obtained by the HS decomposition given by Eq. (18).

Let us formulate the EPR problem by following the “quantum mystery” description presented by Mermin for three entangled particles [17], which holds here also for two entangled particles: “In the absence of connections between the detectors and the source, a particle has no information about how the switch of its detector will be set until it arrives there”. “It would seem to be essential for each particle to be carrying instructions for how its detector should flash for either of two possible switch settings it might find upon arrival”. We would like to show here that QM does not introduce any nonlocality problem. According to QM all the measurements which can be made on the subsystems a and b of the singlet Bohm state are fixed by the “instructions” \vec{r}, \vec{s} and T , 15 parameters with the values $t_{11} = t_{22} = t_{33} = -1$ and the other 12 parameters equal to zero, which were obtained during the production stage of the entangled state. Bell’s inequalities have been refuted by applying different sets of measurements in the different arms of the measuring device [14-17]. Although the representation (18) assumes axes of measurements corresponding to $\sigma_1, \sigma_2, \sigma_3$ (which might be defined as the x, y and z axes) changes of axes of measurements can be obtained by rotation from the basis F to another basis F' [28]:

$$(\vec{\sigma}^{F'})_a = O_1(\vec{\sigma}^F)_a ; \quad (\vec{\sigma}^{F'})_b = O_2(\vec{\sigma}^F)_b \quad (20)$$

The essential point here is that the rotation of axes in system a can be done *independently* of the rotation of axes in system b so that in addition to the “instruction” obtained by the QM interaction each observer can rotate individually his axes

of measurement. As noted in Ref. 23, the assumption of vanishing *local* commutators pertaining to *single* systems which is assumed in local-realistic hidden variables theories, is the one that leads to contradiction with QM. A similar conclusion was reached by Mermin: “This is extremely pleasing, for it is just the fact that the x and y components of the spin of a *single* particle do not commute which leads the well educated quantum mechanician to reject from the start the inference of instruction sets.” In conclusion, “instruction sets” which lead to local-realistic theory are in contradiction with QM, but “instruction sets” obtained by QM which do not violate locality but violate realistic models, even for single particles, can be obtained in the HS decomposition of the density matrix.

For the singlet spin system one can assume a common rotation for the two entangled subsystems and then the density matrix can be expressed in the new frame F' with the same expression of Eq. (16) but with $(\vec{\sigma})^F$ replaced by $(\vec{\sigma})^{F'}$. This kind of symmetry follows from the equality $t_{11} = t_{22} = t_{33}$. For a triplet state with $M = 0$ we get $t_{33} = -1$, $t_{11} = t_{22} = 1$ and Eq. (16) will change its form even if it follows a common rotation $O_1 = O_2$. We find that the symmetry properties and their relation to quantum measurements are described in a simple way by the HS decomposition [28].

For an entangled state of three two-level particles (denoted by a, b, c), the HS decomposition becomes:

$$\begin{aligned}
8\rho = & (I)_a \otimes (I)_b \otimes (I)_c + (\vec{r} \cdot \vec{\sigma})_a \otimes (I)_b \otimes (I)_c + (I)_a \otimes (\vec{s} \cdot \vec{\sigma})_b \otimes (I)_c + \\
& + (I)_a \otimes (I)_b \otimes (\vec{p} \cdot \vec{\sigma})_c + \sum_{mn} t_{mn} (I)_a \otimes (\sigma_m)_b \otimes (\sigma_n)_c \\
& + \sum_{k\ell} o_{k\ell} (\sigma_k)_a \otimes (I)_b \otimes (\sigma_\ell)_c + \sum_{ij} p_{ij} (\sigma_i)_a \otimes (\sigma_j)_b \otimes (I)_c
\end{aligned}$$

$$+ \sum_{\alpha, \beta, \gamma} R_{\alpha\beta\gamma} (\sigma_\alpha)_a \otimes (\sigma_\beta)_b \otimes (\sigma_\gamma)_c \quad (21)$$

We find that the three two-level entangled state is described by 63 parameters: 9 for \vec{r} , \vec{s} and \vec{p} , 27 for t_{mn} , $o_{k\ell}$ and p_{ij} and 27 for $R_{\alpha\beta\gamma}$. The parameters \vec{r} , \vec{s} and \vec{p} are obtained by measurement on one arm of the measurement device, t_{mn} , $o_{k\ell}$ and p_{ij} are obtained by the measurements on the corresponding two arms of the measurement device and $R_{\alpha\beta\gamma}$ are obtained by the corresponding measurements on the three arms of the measuring device. For example the parameters for the entangled state

$$|\psi\rangle = \frac{|1\rangle_a |1\rangle_b |1\rangle_c + |2\rangle_a |2\rangle_b |2\rangle_c}{\sqrt{2}} \quad (22)$$

are given by

$$R_{122} = R_{212} = R_{221} = -1 ; \quad t_{33} = o_{33} = p_{33} = R_{111} = 1$$

and all other parameters are equal to zero. Here again the change of axes of measurement can be made in the systems a, b and c and by independent rotations for $(\vec{\sigma})_a$, $(\vec{\sigma})_b$ and $(\vec{\sigma})_c$, respectively.

The Shannon index of correlation can be given for any number of two-level system by generalizing Eq. (4):

$$I_{\text{Shann}} = \sum_{ijkl\dots} P_{ijkl\dots} \log \left(\frac{P_{ijkl\dots}}{P_i P_j P_k P_\ell \dots} \right) \quad (23)$$

and the quantum correlation for such systems is given by generalizing Eq. (7)

$$I_c = S_a + S_b + S_c + S_d \dots - S \quad (24)$$

where $S = 0$ for pure quantum states. For the canonical representation of n particle GHZ system which are maximally correlated one gets [20,28]:

$$S_a = S_b = S_c = S_d = \dots = \log 2, \quad I_c = n \log 2 \quad (25)$$

while for the Shannon index of correlation defined by Eq. (23) one gets

$$I_{\text{Shann}} = (n - 1) \log 2 \quad (26)$$

Generally, for these canonical representations there is one bit of information less in the Shannon index of correlation relative to that of the quantum correlation.

4. Summary and Conclusions

In the present study we have followed the “orthodox approach”, in which QM describes ensemble averaging, and have related QM results for entangled two-level systems to information theory. We have described various properties of canonical representations which show maximal correlations and have discussed for these states the difference between the Shannon index of correlation and the quantum index of correlation. It was shown that hidden variable theories introduce a refinement of the quantum state leading to a Shannon index of correlation which is larger than that predicted by QM. By refuting local hidden variable theories one puts a certain limit on the information that can be transmitted between subsystems of the entangled state.

By using the HS decomposition of entangled two-level systems we have shown that QM does not violate locality. The parameters of the HS decomposition, which have been fixed during the interaction time, enable us to predict the results of any measurement which will be made on the separated subsystems. The changes of axes of measurement lead to rotations of the spin vectors which can be made independently by the observers in different arms of the measurement device. Violations

of Bell's inequalities or Bell's theorem follow from commutation relations for single particles so that the assumption of realism is violated and not locality. The HS decomposition has been analyzed for some cases including especially the Bohm singlet state and GHZ entangled states.

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